17216

14115

3 Hours/100 Marks

Seat No. | | | | | | | | |

Instructions *: (1)* ***All*** *questions are* ***compulsory****.*
(2) Answer ***each*** *next main question on* ***a new*** *page.*
(3) Figures to the ***right*** *indicate* ***full*** *marks.*
(4) ***Assume*** *suitable data, if* ***necessary****.*
(5) ***Use*** *of Non-programmable Electronic Pocket Calculator is* ***permissible****.*
(6) Mobile Phone, Pager and any other Electronic Communication devices are ***not permissible*** *in Examination Hall.*

MARKS

1. Attempt **any ten** of the following : **20**

a) If $(3x - 4y) + i(x + y) = 7$ find x, y.

b) If $z = 1 + \sqrt{3}\ i$, show that $z^2 + 4 = 2z$.

c) If $f(x) = 3x^2 - 5x + 7$, show that $f(-1) = 3f(1)$.

d) State whether the function $f(x) = \frac{e^x + e^{-x}}{2}$ is odd or even.

e) Evaluate $\lim_{x \to 3} \frac{x^2 - 9}{x - 3}$

f) Evaluate $\lim_{x \to 0} \frac{1 - \cos x}{x^2}$.

g) Evaluate $\lim_{x \to 0} \frac{3^x - 4^x}{x}$.

h) Find $\frac{dy}{dx}$, if $y = \log(x^2 + 2x)$.

P.T.O.

MARKS

i) If $x^2 + y^2 = 4$, find dy/dx.

j) Find dy/dx, if $x = \sin\theta$, $y = \cos\theta$.

k) Show that root of equation $x^3 - 2x - 5 = 0$ lies between 2 and 3.

l) Find the first iteration by using Jacobi's method for the following system of equation.

$10x + y + 2z = 13$, $3x + 10y + z = 14$, $2x + 3y + 10z = 15$.

2. Attempt **any four** of the following : **16**

a) Express the following complex number in polar form $-1/2 + \sqrt{3}/2\, i$.

b) Evaluate $(1 + i)^8 + (1 - i)^8 = 32$.

c) Using Euler's formula prove that $\sin^2\theta + \cos^2\theta = 1$.

d) Simplify using De-Moivres theorem.

$$\frac{(\cos 5\theta - i\sin 5\theta)^{2/5}\ (\cos 2/7\,\theta + i\sin 2/7\,\theta)^7}{(\cos 4\theta + i\sin 4\theta)^{1/4}\ (\cos 2/3\,\theta - i\sin 2/3\,\theta)^3}$$

e) If $y = f(x) = \dfrac{2x-3}{3x-2}$ then prove that $x = f(y)$.

f) If $f(x) = x^2 - 4x + 11$, solve the equation $f(x) = f(3x - 1)$.

3. Attempt **any four** of the following : **16**

a) If $f(x) = \log\left(\dfrac{1+x}{1-x}\right)$ then prove that $f\left(\dfrac{2x}{1+x^2}\right) = 2f(x)$.

b) If $f(x) = \dfrac{1}{1-x}$, show that $f[f\{f(x)\}] = x$.

c) Evaluate $\lim\limits_{x\to 1} \dfrac{x^3 + 3x^2 - 6x + 2}{x^3 + 3x^2 - 3x - 1}$.

MARKS

d) Evaluate $\lim_{x\to\infty}\left[\sqrt{x^2+5x}-x\right]$.

e) Evaluate $\lim_{x\to 0}\frac{6^x-3^x-2^x+1}{x^2}$.

f) Evaluate $\lim_{x\to 0}\frac{\sin 3x-3\sin x}{x^3}$.

4. Attempt **any four** of the following : **16**

a) Using first principle find derivative of f(x) = sinx.

b) If u and v are differentiable functions of x and y = u.v, then prove that

$$\frac{dy}{dx}=u.\frac{dv}{dx}+v.\frac{du}{dx}.$$

c) If $y=\frac{e^x+e^{-x}}{e^x-e^{-x}}$, find $\frac{dy}{dx}$

d) Differentiate w.r.t. x, $\tan^{-1}\left(\frac{5x}{1-6x^2}\right)$.

e) If $y=(\sin x)^{\cos x}$, find $\frac{dy}{dx}$.

f) If $y=\tan^{-1}\left(\frac{2t}{1-t^2}\right)$ and $x=\sin^{-1}\left(\frac{2t}{1+t^2}\right)$, find dy/dx.

5. Attempt **any four** of the following : **16**

a) Evaluate $\lim_{x\to 0}\frac{\tan x\,(5^x-1)}{\left(\sqrt{x^2+16}-4\right)}$.

b) Evaluate $\lim_{x\to 3}\frac{\log x-\log 3}{(x-3)}$.

MARKS

c) Using Bisection method, find the approximate root of $x^3 - 6x + 3 = 0$ (three iteration only).

d) Using Regula Falsi method, find the root of $x^3 - x - 4 = 0$ (three iteration only).

e) Using Newton-Raphson method, find the root of $x^4 - x - 9 = 0$.

f) Using Newton-Raphson method, find the approximate value of $\sqrt{10}$ (three iteration only).

6. Attempt **any four** of the following : **16**

a) If $y = \sin 5x - 3\cos 5x$, show that $\frac{d^2y}{dx^2} + 25y = 0$.

b) If $x = a(\theta - \sin\theta)$ and $y = a(1 - \cos\theta)$ find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{4}$.

c) Solve by Jacobi's method (three iteration only)

$5x + 2y + 7z = 30$, $x + 4y + 2z = 15$

$x + 2y + 5z = 20$

d) Solve by Gauss elimination method

$x + 2y + 3z = 14$, $3x + y + 2z = 11$

$2x + 3y + z = 11$

e) Solve by Jacobi's method (three iteration only)

$20x + y - 2z = 17$, $3x + 20y - z = -18$

$2x - 3y + 20z = 25$.

f) Solve by Gauss – Seidal method (three iteration only)

$15x + 2y + z = 18$, $2x + 20y - 32 = 19$

$3x - 6y + 25z = 22$.

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